# A Spreadsheet Simulation Of The Monty Hall Problem

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**ABSTRACT**

*The "Monty Hall" problem or "Three Door" problem—where a person chooses one of three doors in hope of winning a valuable prize but is subsequently offered the choice of changing his or her selection—is a well known and often discussed probability problem. In this paper, the structure, history, and ultimate solution of the Monty Hall problem are discussed. The problem solution is modeled with a spreadsheet simulation that evaluates the frequencies of the possible outcomes (win or lose) under the two choices or strategies available: switch to the unopened door or do not switch. A Law of Large Numbers approach is also used to graphically demonstrate the long run outcome of adopting one the two available strategies. As is known, the optimal strategy is to switch to the unopened door; the spreadsheet model illustrates why this strategy is optimal. A complete discussion of the spreadsheet logic is included. Pedagogical approaches and applications of the spreadsheet simulation approach are also discussed.*

**Keywords:** spreadsheet, probability, simulation, Law of Large Numbers, Monty Hall problem

## INTRODUCTION

One of the best known and most frequently discussed math\statistics problems is the "Monty Hall" problem. Monty Hall was the legendary host of a television game show "Let's Make a Deal", which debuted on network television from December 31, 1963 to January 3, 1964. Various formats of the popular program appeared over the next 40 years, with the later attempts achieving little popularity compared to the programs aired in the 1960s and 1970s.[1] A key element to the game show involved three doors. Behind each door was a prize. Two of the prizes were of no value, while the third door held a prize of significant value. The participant was asked to choose a door. After the selection of a door, Monty Hall would reveal one of the "no value" prizes behind a door. Since the host knew the location of the "high value" prize, he would never open this door. Nor would he open the door that the contestant had chosen. Monty would then offer the contestant the opportunity to stay with his original choice or to change to the remaining un-opened door. In the various versions of the program that appeared intermittently on television from 1963 until 2003, different approaches to the three-door problem were introduced, including a fourth door in 1984.[2]

The purpose of this paper is to present a spreadsheet simulation model of the "Monty Hall" problem, which can be used to provide insight to the probabilities involved with the problem and help one understand why there is a best answer to the key question in the problem. That key question is "Should the contestant keep his original selection or switch to the other remaining door?" The answer to this question, while simple for some individuals has proven to be difficult and frustrating for others, even those with significant education in mathematics.

## LITERATURE REVIEW

The Monty Hall problem has proven to be a mainstay in the literature of mathematics and statistics for many years. Perhaps the most recent discussion and debate centered on a newspaper column by Marilyn vos Savant, which appeared in September 1990. That question was: "Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others, goats. You pick a door, say 1 and the host, who knows what's behind the doors, opens another door, say No. 3, which has a goat. He then says to you, "Do you want to pick door No. 2?" Is it to your advantage to switch your choice? - Craig F. Whitaker, Columbia, MD." Ms. vos Savant responded that indeed there was a "best" answer. She stated that one should always switch. She went on to say that by switching, one would double the odds of winning the car by switching from the original selection. Her answer set off a flurry of debate and discussion. Ms. vos Savant estimated that she received 10,000 letters and that most disagreed with her. Some of the most critical letters came from mathematicians and scientists.

Noted mathematician Andrew Vazsonyi has written extensively concerning the three door problem. He even titled his autobiography Which Door Has the Cadillac: Adventures of a Real Life Mathematician. In an article published in *Decision Line*, Dr. Vazsonyi discusses his amusement and frustration at the inability of those who should realize that Ms. vos Savant was clearly correct in recommending that switching was the best strategy.[3] A particularly interesting exchange occurred between Vazsonyi and his good friend Paul Erdos. Erdos was "one of the century's greatest mathematicians, who posed and solved thorny problems in number theory and other areas and founded the field of discrete mathematics, which is the foundation of computer science. He was also one of the most prolific mathematicians in history, with more than 1,500 papers to his name."[4] Vazsonyi relates how in 1995, after relating the goats and Cadillac problem and the answer (always switch), Erdos responded "No. That is impossible." Vazsonyi was convinced, along with many others, that decision trees would provide insight and help others to see why the switching strategy was the correct answer. Hammer expanded on the decision tree approach in his paper "A Genuine Decision Tree for the Monty Hall Problem."[5] In both the 1999 paper and a follow-up paper published in 2003, Vazsonyi discussed the utilization of simulation as a solution, as well as the need for a "non-mathematical" explanation. Vos Savant also suggested simulation as an exercise that would be enlightening and convincing.[6] There are numerous interactive programs which have been developed and are available on the internet which simulate the problem. Role playing simulation has also been suggested. Various approaches to a classroom approach to simulating the problem have been advanced by Umble and Umble[7] and Taras and Grossman.[8] Key to the utilization of simulation of this problem is that the sample size of the simulation runs must be sufficient. As will be demonstrated, sometimes a very large number of runs are required before the outcome of the Law of Large Numbers can be observed. Also critical is an understanding of the rules of the game as defined previously. It is possible that a misunderstanding of one or more of the key rules could help explain why so many individuals fail to see why switching is always the better action.[9]

Dr. Vazsonyi attempted to provide a "non-mathematical" solution in 2003. He identified every possible outcome for switching and counted the number of wins and losses. His approach is duplicated in Table 1, with some modifications.[10]

As indicated in Table 1, of the nine possible outcomes, by switching, one will win six times. This is exactly 2 to 1 or a doubling of the probability of winning, as suggested by vos Savant in her newspaper column, as well as by Vazsonyi and many others, mathematicians and non-mathematicians alike.

**Table 1**
**Monty Hall Problem Switching Strategy**

| Case Number | Car Behind | You Guess | Monty Opens | Switch To | Result |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 2 or 3 | 3 or 2 | Lose |
| 2 | 1 | 2 | 3 | 1 | Win |
| 3 | 1 | 3 | 2 | 1 | Win |
| 4 | 2 | 1 | 3 | 2 | Win |
| 5 | 2 | 2 | 1 or 3 | 3 or 1 | Lose |
| 6 | 2 | 3 | 1 | 2 | Win |
| 7 | 3 | 1 | 2 | 3 | Win |
| 8 | 3 | 2 | 1 | 3 | Win |
| 9 | 3 | 3 | 1 or 2 | 2 or 1 | Lose |

In his analysis, Vazsonyi did not see a need to duplicate the "non-mathematician" approach for the not switching options. However, since so much of this problem appears to be counter-intuitive, the non-switching possibilities are presented in Table 2.

**Table 2**
**Monty Hall Problem Non-Switching (Stay) Strategy**

| Case Number | Car Behind | You Guess | Monty Opens | Stay | Result |
|---|---|---|---|---|---|
| 1 | 1 | 1 | 2 or 3 | 1 | Win |
| 2 | 1 | 2 | 3 | 2 | Lose |
| 3 | 1 | 3 | 2 | 3 | Lose |
| 4 | 2 | 1 | 3 | 1 | Lose |
| 5 | 2 | 2 | 1 or 3 | 2 | Win |
| 6 | 2 | 3 | 1 | 3 | Lose |
| 7 | 3 | 1 | 2 | 1 | Lose |
| 8 | 3 | 2 | 1 | 2 | Lose |
| 9 | 3 | 3 | 1 or 2 | 3 | Win |

Tables 1 and 2 clearly show that by switching one will win six out of nine times and by not switching, one will win only three out of nine times. As shown, switching will win if you guessed wrong to begin with, which one will do two-thirds of the time. Not switching will win only if one guesses correctly at first. With a car behind one of the three doors and the other two holding goats, the probability of winning the car in one guess is 1/3.

Another approach is to re-formulate the problem. Suppose that after the contestant has made his selection, the game show host does not open a door. Instead he offers to trade the two other doors, not including the one initially selected, for the one door you chose. Also, one must change to the assumption that the host does **not know** which door holds the car, thus he is not acting maliciously. There is a 1/3 probability that you guessed correctly. Likewise, there is a 2/3 probability that the car is behind one of the other two doors. Would you be willing to trade your one door for the other two? For some individuals, this explanation clarifies the problem.

As previously discussed, a number of interactive simulation approaches have been developed to help individuals understand the Monty Hall problem. Of the numerous simulation programs available on the internet, none of those of which the authors are aware has utilized a spreadsheet approach. The authors have developed an Excel spreadsheet model which simulates the exercise and shows that over the long run, switching doors does indeed double the probability of winning.

## SPREADSHEET SIMULATION MODEL

Utilizing spreadsheets in the field of operations research (OR)\management science is becoming more and more common. A review of OR textbooks published in the past five years clearly indicates a trend toward expanding the spreadsheet approach to model development. A similar trend emphasizing the use of Excel is evident in business statistics textbooks. The spreadsheet discussed in this paper is designed to simulate the classic "Monty Hall" problem discussed previously. Before discussing the spreadsheet, a brief review of the assumptions and the rules of the game is appropriate. Below are the rules and assumptions.

1. There are three doors. At the beginning of the game a prize will be placed behind each door. The three prizes are an expensive car, say a Cadillac, and other prizes of little value, say two goats. The Cadillac will be placed behind a single door, at random. Each of the other two doors will have a goat placed behind it.
2. The contestant will select a door. Again this is done at random. The contestant has no clue as to which door holds the car.

3. The game show host will open a door. However, this is not done at random. The host knows which door holds the Cadillac. He will never select to reveal this door. He also will not reveal the door the contestant has chosen. If the contestant has not chosen the door with the car, the host must choose the remaining door, the one not holding the car and not chosen by the contestant. However, if the contestant has correctly chosen the door holding the car, the host can choose either of the remaining doors, since each holds a goat.
4. After opening one door, the host asks the contestant if he would like to keep his original selection or switch to the remaining un-opened door.
5. The contestant makes a decision – either switch or stay with the original selection.
6. Should the contestant switch or stay with his original selection?

One needs to clearly understand the rules of the game in order to correctly analyze the problem and answer the final question correctly. Perhaps the most misunderstood rule deals with point 3 above. The host does not randomly select a door to open and show the contestant. He knows which door has the Cadillac and which doors hold goats. He will never reveal the car. Nor will he ever open the door initially chosen by the contestant. It is hypothesized by the authors that some individuals, who have not correctly analyzed the problem, may not have understood these assumptions\rules. The spreadsheet sheet was designed to run 99 simulations per sample. The sample size was selected to illustrate the point that as more simulation runs were executed, the long term expected number of wins when switching is 66 and when not switching is 33. As designed, the spreadsheet can be re-calculated as many times as one desires simply by hitting the F9 key. The results from each recalculation are summarized at the top of the sheet. The spreadsheet displayed in Table 3 is presented as a representative simulation result. A description of the spreadsheet cells follows.

**Table 3**
**Monty Hall Spreadsheet Simulation**

| 1/A | B | C | D | E | F | G | H | I | J | K | L | M | N |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2 | | | | | | | | | | | | | |
| 3 | | 0 | 1 | 0 | 2 | 0 | 1 | 0 | 1 | | 1 | 2 | 3 |
| 4 | | 0.333333 | 2 | 0.5 | 3 | 0.5 | 3 | 0.5 | 2 | 1 | | 3 | 2 |
| 5 | | 0.666667 | 3 | Total | Wins | Losses | | | | 2 | 3 | | 1 |
| 6 | | | | | 66 | 33 | | | | 3 | 2 | 1 | |
| 8 | | | | | | | | | | | | | |
| 9 | Sim# | Random1 | Door | Random2 | Choice | Random3 | Out | Revealed | Switch | NoSwitch | SwitchResult | NoSwithchResult | |
| 10 | 1 | 0.498519 | 2 | 0.873056 | 3 | 0.15828 | 1 | 1 | 2 | 3 | Win | Lose | |
| 11 | 2 | 0.764653 | 3 | 0.856996 | 3 | 0.90988 | | 2 | 1 | 3 | Lose | Win | |
| 12 | 3 | 0.692969 | 3 | 0.118061 | 1 | 0.41327 | 2 | 2 | 3 | 1 | Win | Lose | |
| 13 | 4 | 0.091835 | 1 | 0.964655 | 3 | 0.56422 | 2 | 2 | 1 | 3 | Win | Lose | |
| 14 | 5 | 0.413782 | 2 | 0.540291 | 2 | 0.72267 | | 3 | 1 | 2 | Lose | Win | |
| 15 | 6 | 0.292061 | 1 | 0.734964 | 3 | 0.74264 | 2 | 2 | 1 | 3 | Win | Lose | |
| 16 | 7 | 0.132525 | 1 | 0.390584 | 2 | 0.89782 | 3 | 3 | 1 | 2 | Win | Lose | |
| 17 | 8 | 0.407003 | 2 | 0.365258 | 2 | 0.12633 | | 1 | 3 | 2 | Lose | Win | |
| 18 | 9 | 0.334808 | 2 | 0.23675 | 1 | 0.30617 | 3 | 3 | 2 | 1 | Win | Lose | |
| 19 | 10 | 0.35404 | 2 | 0.35283 | 2 | 0.42186 | | 1 | 3 | 2 | Lose | Win | |
| 20 | 11 | 0.958063 | 3 | 0.585047 | 2 | 0.46922 | 1 | 1 | 3 | 2 | Win | Lose | |
| 21 | 12 | 0.657201 | 2 | 0.801389 | 3 | 0.41516 | 1 | 1 | 2 | 3 | Win | Lose | |
| 21 | 13 | 0.994867 | 3 | 0.927888 | 3 | 0.10622 | | 1 | 2 | 3 | Lose | Win | |
| 22 | 14 | 0.044671 | 1 | 0.517344 | 2 | 0.21687 | 3 | 3 | 1 | 2 | Win | Lose | |
| 23 | 15 | 0.416078 | 2 | 0.294186 | 1 | 0.10488 | 3 | 3 | 2 | 1 | Win | Lose | |
| 24 | 16 | 0.062731 | 1 | 0.128485 | 1 | 0.6987 | | 3 | 2 | 1 | Lose | Win | |
| 25 | 17 | 0.337534 | 2 | 0.137677 | 1 | 0.53625 | 3 | 3 | 2 | 1 | Win | Lose | |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 26 | 18 | 0.852655 | 3 | 0.603364 | 2 | 0.32424 | 1 | 1 | 3 | 2 | Win | Lose | |
| 27 | 19 | 0.039751 | 1 | 0.259019 | 1 | 0.11273 | | 2 | 3 | 1 | Lose | Win | |
| 28 | 20 | 0.297734 | 1 | 0.535494 | 2 | 0.26565 | 3 | 3 | 1 | 2 | Win | Lose | |
| 29 | 21 | 0.329226 | 1 | 0.638236 | 2 | 0.90317 | 3 | 3 | 1 | 2 | Win | Lose | |
| 30 | 22 | 0.570759 | 2 | 0.214577 | 1 | 0.70702 | 3 | 3 | 2 | 1 | Win | Lose | |
| 31 | 23 | 0.917422 | 3 | 0.414185 | 2 | 0.49644 | 1 | 1 | 3 | 2 | Win | Lose | |
| 32 | 24 | 0.706466 | 3 | 0.378729 | 2 | 0.74537 | 1 | 1 | 3 | 2 | Win | Lose | |
| 33 | 25 | 0.614415 | 2 | 0.550291 | 2 | 0.07652 | | 1 | 3 | 2 | Lose | Win | |
| 34 | 26 | 0.569528 | 2 | 0.058075 | 1 | 0.9112 | 3 | 3 | 2 | 1 | Win | Lose | |
| 35 | 27 | 0.245988 | 1 | 0.561861 | 2 | 0.60683 | 3 | 3 | 1 | 2 | Win | Lose | |
| 36 | 28 | 0.559755 | 2 | 0.577064 | 2 | 0.5342 | | 3 | 1 | 2 | Lose | Win | |
| 37 | 29 | 0.76666 | 3 | 0.268054 | 1 | 0.91481 | 2 | 2 | 3 | 1 | Win | Lose | |
| 38 | 30 | 0.514108 | 2 | 0.913787 | 3 | 0.93796 | 1 | 1 | 2 | 3 | Win | Lose | |
| 39 | 31 | 0.390154 | 2 | 0.60403 | 2 | 0.9065 | | 3 | 1 | 2 | Lose | Win | |
| 40 | 32 | 0.890886 | 3 | 0.832469 | 3 | 0.26155 | | 1 | 2 | 3 | Lose | Win | |
| 41 | 33 | 0.539723 | 2 | 0.397651 | 2 | 0.02027 | | 1 | 3 | 2 | Lose | Win | |
| 42 | 34 | 0.162324 | 1 | 0.638932 | 2 | 0.2078 | 3 | 3 | 1 | 2 | Win | Lose | |
| 43 | 35 | 0.705063 | 3 | 0.727267 | 3 | 0.77365 | | 2 | 1 | 3 | Lose | Win | |
| 44 | 36 | 0.909227 | 3 | 0.796426 | 3 | 0.50998 | | 2 | 1 | 3 | Lose | Win | |
| 45 | 37 | 0.301862 | 1 | 0.730587 | 3 | 0.74171 | 2 | 2 | 1 | 3 | Win | Lose | |
| 46 | 38 | 0.240762 | 1 | 0.317995 | 1 | 0.39961 | | 2 | 3 | 1 | Lose | Win | |
| 47 | 39 | 0.605754 | 2 | 0.036798 | 1 | 0.96652 | 3 | 3 | 2 | 1 | Win | Lose | |
| 48 | 40 | 0.200878 | 1 | 0.694124 | 3 | 0.64553 | 2 | 2 | 1 | 3 | Win | Lose | |
| 49 | 41 | 0.362843 | 2 | 0.516154 | 2 | 0.42495 | | 1 | 3 | 2 | Lose | Win | |
| 50 | 42 | 0.274545 | 1 | 0.191433 | 1 | 0.91887 | | 3 | 2 | 1 | Lose | Win | |
| 51 | 43 | 0.026065 | 1 | 0.258203 | 1 | 0.20229 | | 2 | 3 | 1 | Lose | Win | |
| 52 | 44 | 0.983306 | 3 | 0.180101 | 1 | 0.28968 | 2 | 2 | 3 | 1 | Win | Lose | |
| 53 | 45 | 0.055287 | 1 | 0.684945 | 3 | 0.46391 | 2 | 2 | 1 | 3 | Win | Lose | |
| 54 | 46 | 0.732255 | 3 | 0.205226 | 1 | 0.05939 | 2 | 2 | 3 | 1 | Win | Lose | |
| 55 | 47 | 0.534365 | 2 | 0.189627 | 1 | 0.77119 | 3 | 3 | 2 | 1 | Win | Lose | |
| 56 | 48 | 0.503714 | 2 | 0.675623 | 3 | 0.60947 | 1 | 1 | 2 | 3 | Win | Lose | |
| 57 | 49 | 0.368034 | 2 | 0.106319 | 1 | 0.78031 | 3 | 3 | 2 | 1 | Win | Lose | |
| 58 | 50 | 0.89837 | 3 | 0.283748 | 1 | 0.87645 | 2 | 2 | 3 | 1 | Win | Lose | |
| 59 | 51 | 0.373085 | 2 | 0.5745 | 2 | 0.4992 | | 1 | 3 | 2 | Lose | Win | |
| 60 | 52 | 0.212439 | 1 | 0.396294 | 2 | 0.06733 | 3 | 3 | 1 | 2 | Win | Lose | |
| 61 | 53 | 0.764198 | 3 | 0.483408 | 2 | 0.80703 | 1 | 1 | 3 | 2 | Win | Lose | |
| 62 | 54 | 0.932519 | 3 | 0.278456 | 1 | 0.56518 | 2 | 2 | 3 | 1 | Win | Lose | |
| 63 | 55 | 0.453321 | 2 | 0.711561 | 3 | 0.55273 | 1 | 1 | 2 | 3 | Win | Lose | |
| 64 | 56 | 0.235872 | 1 | 0.413402 | 2 | 0.75149 | 3 | 3 | 1 | 2 | Win | Lose | |
| 65 | 57 | 0.130964 | 1 | 0.917006 | 3 | 0.03824 | 2 | 2 | 1 | 3 | Win | Lose | |
| 66 | 58 | 0.552969 | 2 | 0.34053 | 2 | 0.54382 | | 3 | 1 | 2 | Lose | Win | |

| | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 67 | 59 | 0.177388 | 1 | 0.978919 | 3 | 0.62237 | 2 | 2 | 1 | 3 | Win | Lose | |
| 68 | 60 | 0.41022 | 2 | 0.395601 | 2 | 0.31192 | | 1 | 3 | 2 | Lose | Win | |
| 69 | 61 | 0.187882 | 1 | 0.803684 | 3 | 0.51075 | 2 | 2 | 1 | 3 | Win | Lose | |
| 70 | 62 | 0.458975 | 2 | 0.916531 | 3 | 0.53501 | 1 | 1 | 2 | 3 | Win | Lose | |
| 71 | 63 | 0.407464 | 2 | 0.721739 | 3 | 0.86522 | 1 | 1 | 2 | 3 | Win | Lose | |
| 72 | 64 | 0.927962 | 3 | 0.305337 | 1 | 0.46825 | 2 | 2 | 3 | 1 | Win | Lose | |
| 73 | 65 | 0.463404 | 2 | 0.551701 | 2 | 0.96602 | | 3 | 1 | 2 | Lose | Win | |
| 74 | 66 | 0.315176 | 1 | 0.133195 | 1 | 0.79562 | | 3 | 2 | 1 | Lose | Win | |
| 75 | 67 | 0.558998 | 2 | 0.595494 | 2 | 0.52088 | | 3 | 1 | 2 | Lose | Win | |
| 76 | 68 | 0.317318 | 1 | 0.100917 | 1 | 0.42657 | | 2 | 3 | 1 | Lose | Win | |
| 77 | 69 | 0.042551 | 1 | 0.733411 | 3 | 0.21813 | 2 | 2 | 1 | 3 | Win | Lose | |
| 78 | 70 | 0.100247 | 1 | 0.446379 | 2 | 0.60676 | 3 | 3 | 1 | 2 | Win | Lose | |
| 79 | 71 | 0.528992 | 2 | 0.288292 | 1 | 0.21926 | 3 | 3 | 2 | 1 | Win | Lose | |
| 80 | 72 | 0.538548 | 2 | 0.571462 | 2 | 0.78424 | | 3 | 1 | 2 | Lose | Win | |
| 81 | 73 | 0.696605 | 3 | 0.674522 | 3 | 0.31408 | | 1 | 2 | 3 | Lose | Win | |
| 82 | 74 | 0.907796 | 3 | 0.069921 | 1 | 0.71033 | 2 | 2 | 3 | 1 | Win | Lose | |
| 83 | 75 | 0.426826 | 2 | 0.962449 | 3 | 0.66607 | 1 | 1 | 2 | 3 | Win | Lose | |
| 84 | 76 | 0.773521 | 3 | 0.452353 | 2 | 0.16357 | 1 | 1 | 3 | 2 | Win | Lose | |
| 85 | 77 | 0.009189 | 1 | 0.484044 | 2 | 0.75196 | 3 | 3 | 1 | 2 | Win | Lose | |
| 86 | 78 | 0.04483 | 1 | 0.41715 | 2 | 0.47709 | 3 | 3 | 1 | 2 | Win | Lose | |
| 87 | 79 | 0.891682 | 3 | 0.363372 | 2 | 0.98516 | 1 | 1 | 3 | 2 | Win | Lose | |
| 88 | 80 | 0.974967 | 3 | 0.415107 | 2 | 0.38028 | 1 | 1 | 3 | 2 | Win | Lose | |
| 89 | 81 | 0.338683 | 2 | 0.351403 | 2 | 0.58173 | | 3 | 1 | 2 | Lose | Win | |
| 90 | 82 | 0.531121 | 2 | 0.030885 | 1 | 0.42637 | 3 | 3 | 2 | 1 | Win | Lose | |
| 91 | 83 | 0.55797 | 2 | 0.886605 | 3 | 0.04737 | 1 | 1 | 2 | 3 | Win | Lose | |
| 92 | 84 | 0.143537 | 1 | 0.451067 | 2 | 0.54158 | 3 | 3 | 1 | 2 | Win | Lose | |
| 93 | 85 | 0.38919 | 2 | 0.854615 | 3 | 0.05858 | 1 | 1 | 2 | 3 | Win | Lose | |
| 94 | 86 | 0.873296 | 3 | 0.280283 | 1 | 0.3639 | 2 | 2 | 3 | 1 | Win | Lose | |
| 95 | 87 | 0.808479 | 3 | 0.776967 | 3 | 0.33532 | | 1 | 2 | 3 | Lose | Win | |
| 96 | 88 | 0.14546 | 1 | 0.885646 | 3 | 0.61248 | 2 | 2 | 1 | 3 | Win | Lose | |
| 97 | 89 | 0.741541 | 3 | 0.750181 | 3 | 0.09647 | | 1 | 2 | 3 | Lose | Win | |
| 98 | 90 | 0.502957 | 2 | 0.937612 | 3 | 0.41848 | 1 | 1 | 2 | 3 | Win | Lose | |
| 99 | 91 | 0.321223 | 1 | 0.210886 | 1 | 0.23594 | | 2 | 3 | 1 | Lose | Win | |
| 100 | 92 | 0.093371 | 1 | 0.378565 | 2 | 0.08564 | 3 | 3 | 1 | 2 | Win | Lose | |
| 101 | 93 | 0.083206 | 1 | 0.992961 | 3 | 0.3414 | 2 | 2 | 1 | 3 | Win | Lose | |
| 102 | 94 | 0.713245 | 3 | 0.017565 | 1 | 0.05744 | 2 | 2 | 3 | 1 | Win | Lose | |
| 103 | 95 | 0.034498 | 1 | 0.2356 | 1 | 0.26004 | | 2 | 3 | 1 | Lose | Win | |
| 104 | 96 | 0.588639 | 2 | 0.073045 | 1 | 0.20681 | 3 | 3 | 2 | 1 | Win | Lose | |
| 105 | 97 | 0.356161 | 2 | 0.505462 | 2 | 0.14774 | | 1 | 3 | 2 | Lose | Win | |
| 106 | 98 | 0.783453 | 3 | 0.372016 | 2 | 0.5031 | 1 | 1 | 3 | 2 | Win | Lose | |
| 107 | 99 | 0.479805 | 2 | 0.770417 | 3 | 0.16753 | 1 | 1 | 2 | 3 | Win | Lose | |
| 108 | | Door | f | Door | f | Door | | f | f | f | 66 | 33 | Wins |
| 109 | | 1 | 33 | 1 | 30 | 1 | 21 | 34 | 35 | 30 | | | |
| 110 | | 2 | 39 | 2 | 38 | 2 | 21 | 30 | 31 | 38 | | | |
| 111 | | 3 | 27 | 3 | 31 | 3 | 24 | 35 | 33 | 31 | | | |

*Cells C3:D5* Random number table to assure that each of the 3 doors has a 1/3 probability of holding the Cadillac and also being selected by the contestant.

*Cells E3:J4* Random number table to assure that if the door selected by the contestant holds the car, then there is a 50/50 random chance of revealing one of the two doors holding goats. If the contestant correctly chooses the door holding the car, the host can reveal either of the two remaining doors since each hold goats.

*Cells K3:N6* Table for selecting the out door; that is, if door 1 holds the car and the contestant initially selects door 2, the out door is door 3. If door 1 holds the car and the contestant initially selects door 3, the out door is 2. If door 2 holds the car and the contestant initially selects door 1, the out door is 3, etc. If the contestant incorrectly guesses the door holding the car, the out door must be revealed.

*Cells F6:G6* Display the number of wins (car) and losses (goat) which have occurred if the contestant switched.

*Cells C108:K110* Display frequency distributions for the number of times each of the 3 doors occur in the respective columns of the main simulation.

*Cells L108:M108* Display the number of wins (car) and losses for switching and not switching respectively.

*Cells C9: M107* Hold the values for the main body of the simulation spreadsheet.

A description of the individual columns is shown below:

*Column B* Simulation number.

*Columns C, E, G* Random numbers between 0 and 1 generated by Excel.

*Column C and F* Door number associated with the Excel-generated number.

*Column H* Out door; that is, the door which must be displayed if the contestant chooses the incorrect door - a door holding a goat instead of a car. This door, which does not hold a car and was not chosen by the contestant, is the only one left to reveal.

*Column I* The door opened by the host. If cell H holds a number, this is the door which must be opened. On the other hand, if cell H is blank, which means the contestant guessed correctly, and is the door that holds the car, the host can choose either of the remaining doors since each holds a goat.

*Column J* The door the contestant chooses if he switches from his original selection.

*Column K* The door the contestant chooses if he stays with his original selection.

*Column L* Game result if the contestant switches. Win indicates the contestant has won the car. Lose indicates the contestant has a goat, not the car.

*Column M* Game result if the contestant does not switch and stays with his original door selection.

As indicated in the spreadsheet in Table 3, for this simulation, when the contestant switches doors, the number of wins is 66 compared to 33 losses. Repeated simulation runs can be executed by entering F9. Sampling differences due to the generation of random numbers will provide variation in the output. However, repeated simulation executions indicate a convergence toward the expected 2:1 ratio associated with switch\no switch. A statistical summary of 100 simulation executions is presented in Table 4. As indicated in the table, one does indeed double the probability of winning the car by switching from the original selection. The average number of wins if one switches is 66 compared to only 33 if one chooses to stay with the original selection.

Frequency distributions for the number of wins for switching and not switching for the 100 simulations are displayed in Table 5. Graphical representations of the frequency distributions are shown in Figures 1 and 2.

**Table 4**
**Statistical Summary 100 Simulation Model Executions**
**Number of Wins (Car Door Selected)**

| | **Switch** | **No Switch** |
|---|---|---|
| **Mean** | 66.09 | 32.91 |
| **Median** | 66 | 33 |
| **Mode** | 66 | 33 |
| **Standard Deviation** | 4.62 | 4.62 |
| **Minimum** | 55 | 21 |
| **Maximum** | 78 | 44 |

**Table 5**
**Frequency Distribution**
**Number of Wins Switching and Not Switching**

| **# wins switching** | **F** | **# wins not switching** | **f** |
|---|---|---|---|
| 55 | 1 | 21 | 2 |
| 56 | 1 | 22 | 0 |
| 57 | 1 | 23 | 0 |
| 58 | 2 | 24 | 2 |
| 59 | 1 | 25 | 3 |
| 60 | 5 | 26 | 3 |
| 61 | 4 | 27 | 4 |
| 62 | 6 | 28 | 2 |
| 63 | 7 | 29 | 4 |
| 64 | 7 | 30 | 7 |
| 65 | 10 | 31 | 8 |
| 66 | 10 | 32 | 10 |
| 67 | 10 | 33 | 10 |
| 68 | 8 | 34 | 10 |
| 69 | 7 | 35 | 7 |
| 70 | 4 | 36 | 7 |
| 71 | 2 | 37 | 6 |
| 72 | 4 | 38 | 4 |
| 73 | 3 | 39 | 5 |
| 74 | 3 | 40 | 1 |
| 75 | 2 | 41 | 2 |
| 76 | 0 | 42 | 1 |
| 77 | 0 | 43 | 1 |
| 78 | 2 | 44 | 1 |
| Total | 100 | | 100 |

The other sheets in the worksheet are used in a Law of Large Numbers (LLN) approach. The LLN states that "if a random phenomenon with numerical outcomes is repeated many times independently, the mean value of the actually observed outcomes approaches the expected value."[11] Some business statistics textbooks include discussions of the LLN; for instance, Keller and Warrack describe the LLN in terms of binomial probabilities: "in the long run, the sample proportion will be quite close to the population proportion."[12] The sheet titled 'large numbers switch' uses the logic previously explained to calculate the probability of winning when using the switching strategy. Each trial where the strategy wins is scored as a '1' while each trial where the strategy loses is scored as a '0.' As trials are added, a running average of the scores is computed. There are three macros that modify the sheet, located in column O. The first button activates the first macro that adds a single trial. The second button activates the second macro that compiles 1000 trials. The final button activates the third macro that clears the work area of all but the first trial (required for smooth functioning of the macros). According to the LLN, as more trials are added, the frequencies should behave more like probabilities; thus, the overall average should be the probability of winning under the strategy being considered, i.e., switch or no switch. A plot of the running average is shown on the sheet titled ‘chart “switch”’. As expected, as the number of trials increases, the average approaches 0.66. A similar result is found in the sheets ‘large numbers No Switch’ and ‘chart “no switch”’ for the not switching strategy. A surprisingly large number of trials can be required before the frequencies ‘settle down.’ The charts were configured using the Line Chart type of graph and 1000 trials. This can be easily customized depending on the number of trials desired.

**TEACHING APPLICATIONS**

The simulation has been used in the classroom as a teaching tool. Clearly 99 simulations for this environment is excessive. Instead, the first 15 simulation executions have proven to be a sufficient number to make the key points of the exercise. Four handouts were used in the classroom simulation. First, the column of random numbers and the associated door selected by the contestant is provided for each student. Second, a PowerPoint graphical handout showing the 15 end-of-simulation executions is given to one student. This is done to prove that the instructor is not cheating by moving goats and cars to force a particular result. Third, a complete PowerPoint graphical printout of the first 15 simulations is provided. However, to prevent students from looking ahead in the exercise, this is only done after the classroom exercise is complete. Likewise, after the exercise is complete, the complete printout shown in Table 3 is provided for students. Table 6 shows the random numbers and the door selected values from Table 3. Figure 3 displays the first three end-of-simulation PowerPoint slides. Figure 4 displays the complete PowerPoint slides for the first three simulation executions.

**Table 6**
**Random Numbers and Selected Door**

| Random2 | Choice |
|---|---|
| 0.873056 | 3 |
| 0.856996 | 3 |
| 0.118061 | 1 |
| 0.964655 | 3 |
| 0.540291 | 2 |
| 0.734964 | 3 |
| 0.390584 | 2 |
| 0.365258 | 2 |
| 0.23675 | 1 |
| 0.35283 | 2 |
| 0.585047 | 2 |
| 0.801389 | 3 |
| 0.927888 | 3 |
| 0.517344 | 2 |
| 0.294186 | 1 |

**Figure 3**
**Graphical Output End-of-Simulation Results Simulations 1,2,3**

**Figure 4**
**Graphical Output Simulation Results Simulations 1, 2, 3**

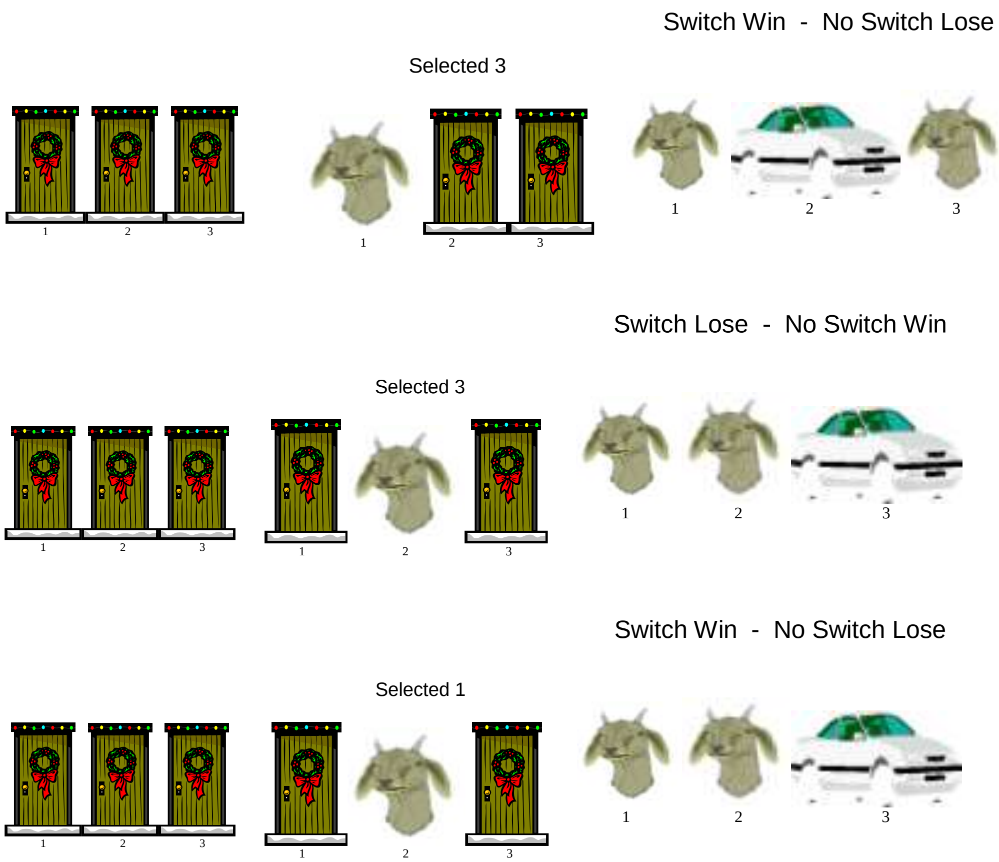
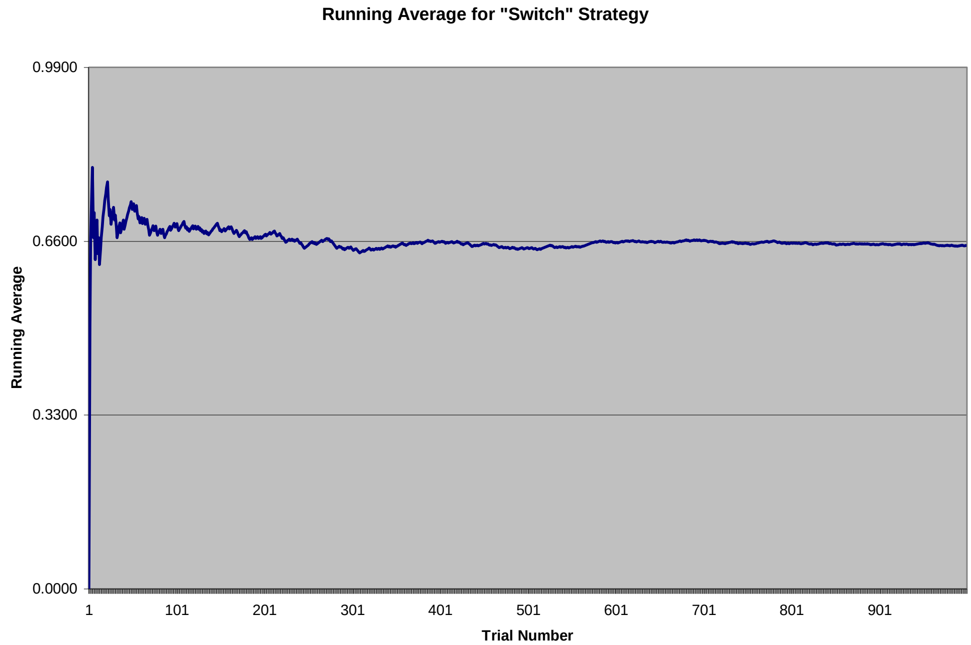



Figure 5 shows the average proportion of successes from 1000 trials using the switch strategy. The LLN sheets can be used in a variety of ways. One approach is to show how long it takes, i.e., how many runs are required for the (empirical) frequencies to resemble (a priori) probabilities. Business statistics students are often surprised when confronted with the notion of the 'long run.' This notion can be extended to discuss games of chance where the number of possible outcomes is large, e.g., state lottery type games. Another use of the spreadsheet solution is the discussion of spreadsheet random numbers. The use of random numbers in the subject spreadsheet is straightforward and can be used to demonstrate the possibilities of simple random sampling using a statistical frame and spreadsheet tools. Having students describe the logic of the spreadsheet simulation may be a useful exercise. A course in decision support or advanced spreadsheet programming instructs students to "develop a spreadsheet to demonstrate the Monty Hall problem."

**Figure 5**




## SUMMARY

The spreadsheet simulation model presented in this paper is intended to help provide insight into the classic Monty Hall problem and provide an alternative approach to utilize in teaching the associated probability principles. Numerous approaches have been utilized to explain why switching doors will double the probability of winning after seeing a door, not the one with the car and not the one initially selected by the contestant. For some, the probability or decision tree is the preferred tool for analysis. Others have utilized role playing in a classroom environment. The discussion and debate generated by the counter-intuitive correct solution continues to this day. It is worth noting that at least one professional mathematician who initially attacked Ms. vos Savant had the courage to admit his error. Robert Sachs of George Mason University had initially challenged vos Savant and said that she was incorrect by writing her, "I'm very concerned with the general public's lack of mathematical skills. Please help by confessing your error." After realizing that vos Savant was indeed correct and he was wrong, he communicated with her. "I wrote her another letter telling her that after removing my foot from my mouth, I'm now eating humble pie. I vowed as a penance to answer all the people who wrote to castigate me. It's been an intense professional embarrassment."[13] Perhaps a misunderstanding of the assumptions and rules, as previously discussed, is a partial explanation of the inability of individuals to grasp the problem. Perhaps it is just that the correct answer is counter-intuitive. As vos Savant stated, "When reality clashes so violently with intuition people are shaken."[14] Such is often the case with business statistics students, especially those who rely on intuitive solutions and formulations for problems involving probabilities (e.g. are these events independent?). Determining the probability of duplicate birthdays in a classroom of students—an often used classroom demonstration[15]—also tends to be counter-intuitive. It is the hope of the authors that the spreadsheet simulation model approach outlined in this paper will provide an alternative approach for both teaching and understanding the classic probability problem known as the Monty Hall problem.

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**NOTES**